

# wave function

- [SQA] 1. Express  $8 \cos x^\circ - 6 \sin x^\circ$  in the form  $k \cos(x^\circ + a^\circ)$  where  $k > 0$  and  $0 < a < 360$ . 4

Part	Marks	Level	Calc.	Content	Answer	U3 OC4	
	4	C	CR	T13	$10 \cos(x^\circ + 36.9^\circ)$	2001 P2 Q5	
				<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: expand <math>k \cos(x^\circ + a^\circ)</math></li> <li>•<sup>2</sup> ic: compare coefficients</li> <li>•<sup>3</sup> pd: process</li> <li>•<sup>4</sup> pd: process</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k \cos x \cos a - k \sin x \sin a</math> <i>stated explicitly</i></li> <li>•<sup>2</sup> <math>k \cos a = 8</math> <b>and</b> <math>k \sin a = 6</math> <i>stated explicitly</i></li> <li>•<sup>3</sup> <math>k = 10</math></li> <li>•<sup>4</sup> <math>a = 36.9</math></li> </ul>		

- [SQA] 2. (a) Write  $\sin(x) - \cos(x)$  in the form  $k \sin(x - a)$  stating the values of  $k$  and  $a$  where  $k > 0$  and  $0 \leq a \leq 2\pi$  4
- (b) Sketch the graph of  $y = \sin(x) - \cos(x)$  for  $0 \leq x \leq 2\pi$ , showing clearly the graph's maximum and minimum values and where it cuts the  $x$ -axis and the  $y$ -axis. 3

Part	Marks	Level	Calc.	Content	Answer	U3 OC4	
(a)	4	C	NC	T13	$\sqrt{2} \sin(x - \frac{\pi}{4})$	2002 P1 Q9	
(b)	3	C	NC	T15, T14	sketch		
				<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: know to expand, and expand</li> <li>•<sup>2</sup> ic: compare coefficients</li> <li>•<sup>3</sup> pd: write down the value of <math>k</math></li> <li>•<sup>4</sup> pd: process <math>a</math></li> <li>•<sup>5</sup> ic: sketch a sine curve</li> <li>•<sup>6</sup> ic: int/com max. and min. values</li> <li>•<sup>7</sup> pd: process intercepts</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k \sin x \cos a - k \cos x \sin a</math> <i>stated explicitly</i></li> <li>•<sup>2</sup> <math>k \cos a = 1</math> <b>and</b> <math>k \sin a = 1</math> <i>stated explicitly</i></li> <li>•<sup>3</sup> <math>k = \sqrt{2}</math></li> <li>•<sup>4</sup> <math>a = \frac{\pi}{4}</math> <i>accept in degrees</i></li> <li>•<sup>5</sup> correct shape of graph (i.e. sin) but not passing through the origin</li> <li>•<sup>6</sup> graph lies between <math>\sqrt{2}</math> and <math>-\sqrt{2}</math></li> <li>•<sup>7</sup> <math>(\frac{\pi}{4}, 0), (\frac{5\pi}{4}, 0), (0, -1)</math> <i>accept only answers in radians</i></li> </ul>		

3. (a)  $12 \cos x^\circ - 5 \sin x^\circ$  can be expressed in the form  $k \cos(x + a)^\circ$ , where  $k > 0$  and  $0 \leq a < 360$ .

Calculate the values of  $k$  and  $a$ .

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- (b) (i) Hence state the maximum and minimum values of  $12 \cos x^\circ - 5 \sin x^\circ$ .  
 (ii) Determine the values of  $x$ , in the interval  $0 \leq x < 360$ , at which these maximum and minimum values occur.

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CN	T13	$k = 13, a = 22.6$	2010 P2 Q2
(bi)	1	C	CN	T14	max 13, min -13	
(bii)	2	C	CN	T14	max at 337.4, min at 157.4	

<ul style="list-style-type: none"> <li>•<sup>1</sup> ss: use addition formula</li> <li>•<sup>2</sup> ic: compare coefficients</li> <li>•<sup>3</sup> pd: process <math>k</math></li> <li>•<sup>4</sup> pd: process <math>a</math></li> <li>•<sup>5</sup> ss: state maximum and minimum</li> <li>•<sup>6</sup> ic: find <math>x</math> corresponding to max. value</li> <li>•<sup>7</sup> pd: find <math>x</math> corresponding to min. value</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k \cos x^\circ \cos a^\circ - k \sin x^\circ \sin a^\circ</math></li> <li>•<sup>2</sup> <math>k \cos a^\circ = 12</math> and <math>k \sin a^\circ = 5</math></li> <li>•<sup>3</sup> 13 (do not accept <math>\sqrt{169}</math>)</li> <li>•<sup>4</sup> 22.6 (accept any answer which rounds to 23)</li> <li>•<sup>5</sup> 13, -13</li> <li>•<sup>6</sup> maximum at 337.4 and no others</li> <li>•<sup>7</sup> minimum at 157.4 and no others</li> </ul>
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[SQA]

4. (a) Express  $\sin x^\circ - 3 \cos x^\circ$  in the form  $k \sin(x - a)^\circ$  where  $k > 0$  and  $0 \leq a < 360$ . Find the values of  $k$  and  $a$ .

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- (b) Find the maximum value of  $5 + \sin x^\circ - 3 \cos x^\circ$  and state a value of  $x$  for which this maximum occurs.

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Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	T13		1992 P1 Q7
(b)	1	C	CR	T14		
(b)	1	A/B	CR	T14		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k \cos a = 1</math></li> <li>•<sup>2</sup> <math>k \sin a = 3</math></li> <li>•<sup>3</sup> <math>k = \sqrt{10}</math></li> <li>•<sup>4</sup> <math>a = 71.6</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>5</sup> maximum = <math>5 + \sqrt{10}</math></li> <li>•<sup>6</sup> angle = <math>161.6^\circ</math></li> </ul>
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[SQA] 5. Solve the simultaneous equations

$$k \sin x^\circ = 5$$

$$k \cos x^\circ = 2,$$

where  $k \geq 0$  and  $0 \leq x \leq 360$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC4		
	4	C	CR	T13	$k = \sqrt{29}, x = 68.2$	1995 P1 Q13		
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; vertical-align: top; border-right: 1px dashed black;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\tan x = \frac{5}{2}</math></li> <li>•<sup>2</sup> <math>x = 68.2</math></li> <li>•<sup>3</sup> <math>k^2 = 25 + 4</math> or <math>k = \frac{5}{\sin 68.2}</math></li> <li>•<sup>4</sup> <math>k = \sqrt{29}</math></li> </ul> </td> <td style="width: 50%; vertical-align: top;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k^2(\sin^2 x + \cos^2 x) = 29</math></li> <li>•<sup>2</sup> <math>k = \sqrt{29}</math></li> <li>•<sup>3</sup> <math>\tan x = \frac{5}{2}</math> or <math>\sin x = \frac{5}{\sqrt{29}}</math></li> <li>•<sup>4</sup> <math>x = 68.2</math></li> </ul> </td> </tr> </table>							<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\tan x = \frac{5}{2}</math></li> <li>•<sup>2</sup> <math>x = 68.2</math></li> <li>•<sup>3</sup> <math>k^2 = 25 + 4</math> or <math>k = \frac{5}{\sin 68.2}</math></li> <li>•<sup>4</sup> <math>k = \sqrt{29}</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k^2(\sin^2 x + \cos^2 x) = 29</math></li> <li>•<sup>2</sup> <math>k = \sqrt{29}</math></li> <li>•<sup>3</sup> <math>\tan x = \frac{5}{2}</math> or <math>\sin x = \frac{5}{\sqrt{29}}</math></li> <li>•<sup>4</sup> <math>x = 68.2</math></li> </ul>
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\tan x = \frac{5}{2}</math></li> <li>•<sup>2</sup> <math>x = 68.2</math></li> <li>•<sup>3</sup> <math>k^2 = 25 + 4</math> or <math>k = \frac{5}{\sin 68.2}</math></li> <li>•<sup>4</sup> <math>k = \sqrt{29}</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k^2(\sin^2 x + \cos^2 x) = 29</math></li> <li>•<sup>2</sup> <math>k = \sqrt{29}</math></li> <li>•<sup>3</sup> <math>\tan x = \frac{5}{2}</math> or <math>\sin x = \frac{5}{\sqrt{29}}</math></li> <li>•<sup>4</sup> <math>x = 68.2</math></li> </ul>							

[SQA] 6. Express  $2 \sin x^\circ - 5 \cos x^\circ$  in the form  $k \sin(x - \alpha)^\circ$ ,  $0 \leq \alpha < 360$  and  $k > 0$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC4
	4	C	CR	T13	$k = \sqrt{29}, \alpha = 68.2$	1997 P1 Q11
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k \sin(x - a) = k \sin x \cos a - k \cos x \sin a</math> <b>stated explicitly</b></li> <li>•<sup>2</sup> <math>k \cos a = 2</math> and <math>k \sin a = 5</math></li> <li>•<sup>3</sup> <math>k = \sqrt{29}</math></li> <li>•<sup>4</sup> <math>a = 68.2</math></li> </ul>						

7. The formula  $d = 200 + 80(\cos 30t^\circ + \sqrt{3} \sin 30t^\circ)$  gives an approximation to the depth of water,  $d$ , measured in centimetres, in a harbour  $t$  hours after midnight.
- (a) Express  $f(t) = \cos 30t^\circ + \sqrt{3} \sin 30t^\circ$  in the form  $k \cos(30t - \alpha)^\circ$  and state the values of  $k$  and  $\alpha$ , where  $0 \leq \alpha \leq 360$ . (4)
- (b) (i) Use your result from part (a) to help you sketch the graph of  $f(t)$  for  $0 \leq t \leq 12$ . (6)
- (ii) Hence, on a separate diagram, sketch the graph of  $d$  for  $0 \leq t \leq 12$ . (6)
- (c) What is the "low-water" time at the harbour during the time interval shown on your graph? (1)
- (d) If the local fishing fleet needs at least 1.5 metres depth of water to enter the harbour without risk of running aground, between what hours must it avoid entering the harbour during the time interval shown on your graph? (2)

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	T13		1989 P2 Q9
(b)	2	C	CR	T1, A3		
(b)	4	A/B	CR	T1, A3		
(c)	1	A/B	CR	CGD		
(d)	2	A/B	CR	CGD		

- (a) •<sup>1</sup>  $k \cos 30t^\circ \cos \alpha^\circ + k \sin 30t^\circ \sin \alpha^\circ$   
 •<sup>2</sup>  $k \cos \alpha^\circ = 1$  and  $k \sin \alpha^\circ = \sqrt{3}$   
 •<sup>3</sup>  $k = 2$   
 •<sup>4</sup>  $\alpha = 60$
- (b) •<sup>5</sup> maximum at (2, 2)  
 •<sup>6</sup> minimum at (8, -2)  
 •<sup>7</sup> endpoints: (0, 1) or (12, 1)  
 •<sup>8</sup> graph correctly annotated with 3 points  
 •<sup>9</sup> sketch with original amplitude increased by factor of 60  
 •<sup>10</sup> sketch with original graph translated  $\begin{pmatrix} 0 \\ 200 \end{pmatrix}$
- (c) •<sup>11</sup> 0800 hours
- (d) •<sup>12</sup> 5.6 hours and 10.4 hours  
 •<sup>13</sup> e.g. between 5am and 11am

- (a) Show that  $2 \cos(x^\circ + 30^\circ) - \sin x^\circ$  can be written as  $\sqrt{3} \cos x^\circ - 2 \sin x^\circ$ . 3
- (b) Express  $\sqrt{3} \cos x^\circ - 2 \sin x^\circ$  in the form  $k \cos(x^\circ + \alpha^\circ)$  where  $k > 0$  and  $0 \leq \alpha \leq 360$  and find the values of  $k$  and  $\alpha$ . 4
- (c) Hence, or otherwise, solve the equation  $2 \cos(x^\circ + 30^\circ) = \sin x^\circ + 1$ ,  $0 \leq x \leq 360$ . 3

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	3	C	CR	T8, T3		1990 P2 Q5
(b)	4	C	CR	T13		
(c)	3	A/B	CR	T16		

(a)  $\bullet^1 \cos(x + 30)^\circ = \cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ$

$\bullet^2 \frac{\sqrt{3}}{2} \cos x^\circ - \frac{1}{2} \sin x^\circ$

$\bullet^3 2 \times \left( \frac{\sqrt{3}}{2} \cos x^\circ - \frac{1}{2} \sin x^\circ \right) - \sin x^\circ$

(b)  $\bullet^4 k \cos x^\circ \cos \alpha^\circ - k \sin x^\circ \sin \alpha^\circ$

$\bullet^5 k \sin \alpha^\circ = \sqrt{3}$  and  $k \sin \alpha^\circ = 1$

$\bullet^6 k = \sqrt{7} \vec{OG} = 426$

$\bullet^7 \alpha = 49.1$

(c)  $\bullet^8 \sqrt{7} \cos(x + 49.1)^\circ = 1$

$\bullet^9 x = 18.7^\circ$

$\bullet^{10} x = 243.1^\circ$

[SQA]

9. The displacement,  $d$  units, of a wave after  $t$  seconds, is given by the formula

$$d = \cos 20t^\circ + \sqrt{3} \sin 20t^\circ.$$

- (a) Express  $d$  in the form  $k \cos(20t^\circ - \alpha^\circ)$ , where  $k > 0$  and  $0 \leq \alpha \leq 360$ . 4
- (b) Sketch the graph of  $d$  for  $0 \leq t \leq 18$ . 4
- (c) Find, correct to one decimal place, the values of  $t$ ,  $0 \leq t \leq 18$ , for which the displacement is 1.5 units. 3

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	T13		1991 P2 Q8
(b)	2	C	CR	T1		
(b)	2	A/B	CR	T1		
(c)	1	C	CR	T7		
(c)	2	A/B	CR	T7		

- (a)
- <sup>1</sup>  $k \cos 20t^\circ \cos \alpha^\circ + k \sin 20t^\circ \sin \alpha^\circ$
  - <sup>2</sup>  $k \cos \alpha^\circ = 1$  and  $k \sin \alpha^\circ = \sqrt{3}$
  - <sup>3</sup>  $k = 2$
  - <sup>4</sup>  $\alpha = 60$
- (b)
- <sup>5</sup> endpoints: (0,1) or (18,1)
  - <sup>6</sup> zeros: (7.5,0) and (16.5,0)
  - <sup>7</sup> stationary points: (3,2) and (12,-2)
  - <sup>8</sup> correct annotation of graph
- (c)
- <sup>9</sup>  $2 \cos(20t - 60)^\circ = 1.5$
  - <sup>10</sup>  $20t - 60 = 41.4 \Rightarrow t = 5.1$
  - <sup>11</sup>  $20t - 60 = -41.4 \Rightarrow t = 0.9$

- [SQA] 10. (a) Express  $3\sin x^\circ - \cos x^\circ$  in the form  $k\sin(x - \alpha)^\circ$ , where  $k > 0$  and  $0 \leq \alpha \leq 90$ . (4)
- (b) Hence find algebraically the values of  $x$  between 0 and 180 for which  $3\sin x^\circ - \cos x^\circ = \sqrt{5}$ . (4)
- (c) Find the range of values of  $x$  between 0 and 180 for which  $3\sin x^\circ - \cos x^\circ \leq \sqrt{5}$ . (2)

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	T13		1994 P2 Q5
(b)	4	C	CR	T16		
(c)	2	A/B	CR	T16		

(a) •<sup>1</sup>  $k(\sin x \cos \alpha - \cos x \sin \alpha)$  or equivalent

•<sup>2</sup>  $k \cos \alpha = 3$  and  $k \sin \alpha = 1$

•<sup>3</sup>  $k = \sqrt{10}$

•<sup>4</sup>  $\alpha = 18.4$

(b) •<sup>5</sup>  $\sqrt{10} \sin(x - 18.4)^\circ = \sqrt{5}$

•<sup>6</sup>  $\sin(x - 18.4)^\circ = \frac{1}{\sqrt{2}}$  or equivalent

•<sup>7</sup>  $63.4$

•<sup>8</sup>  $153.4$

(c) •<sup>9</sup> strategy stated or implied

•<sup>10</sup>  $x \leq 63.4$  and  $x \geq 153.4$

[SQA] 11.  $f(x) = 2\cos x^\circ + 3\sin x^\circ$ .

(a) Express  $f(x)$  in the form  $k\cos(x - \alpha)^\circ$  where  $k > 0$  and  $0 \leq \alpha < 360$ . (4)

(b) Hence solve algebraically  $f(x) = 0.5$  for  $0 \leq x < 360$ . (3)

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	T13		1996 P2 Q7
(b)	3	C	CR	T16		

(a)	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>k \cos x \cos \alpha + k \sin x \sin \alpha</math></li> <li>•<sup>2</sup> <math>k \cos \alpha = 2</math> and <math>k \sin \alpha = 3</math></li> <li>•<sup>3</sup> <math>k = \sqrt{13}</math></li> <li>•<sup>4</sup> <math>\alpha = 56.3</math></li> </ul>
(b)	<ul style="list-style-type: none"> <li>•<sup>5</sup> <math>\cos(x - 56.3)^\circ = \frac{0.5}{\sqrt{13}}</math></li> <li>•<sup>6</sup> <math>x - 56.3 = 82.0, 278.0</math></li> <li>•<sup>7</sup> <math>x = 138.3, 334.3</math></li> </ul>



- [SQA] 12. The function  $f$  is defined by  $f(x) = 2 \cos x^\circ - 3 \sin x^\circ$ .
- (a) Show that  $f(x)$  can be expressed in the form  $f(x) = k \cos(x + \alpha)^\circ$  where  $k > 0$  and  $0 \leq \alpha < 360$ , and determine the values of  $k$  and  $\alpha$ . (4)
- (b) Hence find the maximum and minimum values of  $f(x)$  and the values of  $x$  at which they occur, where  $x$  lies in the interval  $0 \leq x < 360$ . (4)
- (c) Write down the minimum value of  $(f(x))^2$ . (1)

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(a)	4	C	CR	T13		1998 P2 Q7
(b)	1	C	CR	T14		
(b)	3	A/B	CR	T14		
(c)	1	A/B	CR	A6		

- (a)
- <sup>1</sup>  $k \cos x \cos \alpha - k \sin x \sin \alpha$  **stated explicitly**
  - <sup>2</sup>  $k \sin \alpha = 3$  and  $k \cos \alpha = 2$  **stated explicitly**
  - <sup>3</sup>  $k = \sqrt{13}$
  - <sup>4</sup>  $\alpha = 56.3$
- (b)
- <sup>5</sup>  $\sqrt{13} \cos(x + 56.3)$
  - <sup>6</sup> Max =  $\sqrt{13}$  and min =  $-\sqrt{13}$
  - <sup>7</sup>  $x = 303.7$  and no further answers
  - <sup>8</sup>  $x = 123.7$  and no further answers
- (c)
- <sup>9</sup> Min Value = 0

[END OF QUESTIONS]